

# CBCS SCHEME

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15MAT41

## Fourth Semester B.E. Degree Examination, Jan./Feb. 2021 Engineering Mathematics - IV

Time: 3 hrs.

Max. Marks: 80

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. Use of Statistical table is allowed.

### Module-1

- 1 a. Employ Taylor's Series Method to find 'y' at  $x = 0.2$ . Given the linear differential equation  $\frac{dy}{dx} = 3e^x + 2y$  and  $y = 0$  at  $x = 0$  initially considering the terms upto the third degree. (05 Marks)
- b. Use fourth order Runge - Kutta method to solve  $(x + y) \frac{dy}{dx} = 1$ ,  $y(0.4) = 1$  at  $x = 0.5$  correct to four decimal places (Take  $h = 0.1$ ). (05 Marks)
- c. Apply Adams - Bash fourth method to solve  $\frac{dy}{dx} = x^2(1 + y)$ , given that  $y(1) = 1$ ,  $y(1.1) = 1.233$ ,  $y(1.2) = 1.548$  and  $y(1.3) = 1.979$  to evaluate  $y(1.4)$ . (06 Marks)

OR

- 2 a. Given  $\frac{dy}{dx} = x^2 + y$ ,  $y(0) = 1$ . Find correct to four decimal places  $y(0.1)$  using modified Euler's method taking  $h = 0.05$ . (06 Marks)
- b. Use Milne's Predictor and Corrector method to compute  $y$  at  $x = 0.4$ , given  $\frac{dy}{dx} = 2e^x - y$  and
- |   |   |       |       |       |
|---|---|-------|-------|-------|
| x | 0 | 0.1   | 0.2   | 0.3   |
| y | 2 | 2.010 | 2.040 | 2.090 |
- (05 Marks)
- c. Use Fourth order Runge - Kutta method to find  $y(1.1)$ , given  $\frac{dy}{dx} + y - 2x = 0$ ,  $y(1) = 3$  with step size  $h = 0.1$ . (05 Marks)

### Module-2

- 3 a. Given  $\frac{d^2y}{dx^2} - x \frac{dy}{dx} - y = 0$  with the initial conditions  $y(0) = 1$ ,  $y'(0) = 0$ . Compute  $y(0.2)$  using Runge - Kutta method. (05 Marks)
- b. Show that  $J_{\frac{1}{2}}(1) = \sqrt{\frac{2}{\pi x}} \sin x$ . (05 Marks)
- c. Derive Rodrigue's formula  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ . (06 Marks)

OR

- 4 a. Apply Milne's method to compute  $y(0.8)$ . Given that  $\frac{d^2y}{dx^2} = 2y \frac{dy}{dx}$  and the following table of initial values. (05 Marks)

x	0	0.2	0.4	0.6
y	0	0.2027	0.4228	0.6841
y'	1	1.041	1.179	1.468

- b. Express  $f(x) = x^3 + 2x^2 - 4x + 5$  in terms of Legendre Polynomials. (05 Marks)
- c. Show that  $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$ , if  $\alpha \neq \beta$ . Where  $\alpha, \beta$  are roots of  $J_n(x) = 0$ . (06 Marks)

### Module-3

- 5 a. Derive Cauchy - Riemann equations in Cartesian form. (05 Marks)
- b. Using Cauchy's Residue theorem, evaluate the integral  $\int_C \frac{ze^z}{z^2 - 1} dz$ , where  $C$  is the circle  $|Z| = 2$ . (05 Marks)
- c. Find the Bilinear transformation that transforms the points  $Z_1 = 0, Z_2 = 1, Z_3 = \infty$  into the points  $W_1 = -5, W_2 = -1, W_3 = 3$  respectively. (06 Marks)

OR

- 6 a. State and prove Cauchy's theorem. (05 Marks)
- b. Evaluate  $\int_C \frac{\sin^2 Z}{(Z - \pi/6)^3} dz$ , where ' $C$ ' is the circle  $|Z| = 1$ , using Cauchy's integral formula. (05 Marks)
- c. Construct the analytic function whose real part is  $x + e^x \cos y$ . (06 Marks)

### Module-4

- 7 a. Obtain Mean and Variance of Exponential distribution. (05 Marks)
- b. Find the binomial probability distribution which has mean 2 and variance  $\frac{4}{3}$ . (05 Marks)
- c. The Joint probabilities distribution for two Random Variations  $X$  and  $Y$  as follows :

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find i) Marginal distributions of  $X$  and  $Y$  ii) Co-variance of  $X$  and  $Y$ . Also verify that  $X$  and  $Y$  are independent iii) Correlation of  $X$  and  $Y$ . (06 Marks)

OR

- 8 a. A certain number of articles manufactured in one batch were classified into three categories according to a particular characteristic, being less than 50, between 50 and 60 and greater than 60. If this characteristic is known to be normally distributed, determine the mean and standard deviation for this batch if 60%, 35% and 5% were found in these categories.  $[\phi(0.25) = 0.0987, \phi(1.65) = 0.4505]$ . (05 Marks)
- b. Obtain the mean and standard deviation of Poisson distribution. (05 Marks)



- c. Define Random variable. The pdf of a variate  $X$  is given by the following table :

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

- i) Find  $K$ , if this represents a valid probability distribution.  
 ii) Find  $P(x \geq 5)$  and  $P(3 < x \leq 6)$ .

(06 Marks)

**Module-5**

- 9 a. Coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit [ $\chi^2_{0.05} = 9.49$  for 4 d.f].

(06 Marks)

Number of heads	0	1	2	3	4
Frequency	5	29	36	25	5

- b. Find a Unique fixed Probability vector for the regular stochastic matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

(05 Marks)

- c. A group of boys and girls were given an intelligence test. The mean score, S.D score and numbers in each group are as follows :

	Boys	Girls
Mean	74	70
SD	8	10
n	12	10

Is the difference between the means of the two groups significant at 5% level of significance [ $t_{0.05} = 2.086$  for 20 d.f].

(05 Marks)

**OR**

- 10 a. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. (05 Marks)
- b. The weight of 1500 ball bearings are normally distributed with a mean of 635 gms and S.D of 1.36 gms. If 300 random samples of size 36 are drawn from this populations. Determine the expected mean and S.D of the sampling distribution of means if sampling is done  
 i) With replacement ii) without replacement. (05 Marks)
- c. Every year, a man trades his car for a new car. If he has a Maruti, he trades it for an Ambassador. If he has an Ambassador, he trades it for a Santro. However, if he has a Santro, he is just as likely to trade it for a new Santro as a trade it for a Maruti or an Ambassador. In 2000 he bought his first car which was a Santro. Find the probability that he has  
 i) 2002 Santro ii) 2002 Maruti. (06 Marks)

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# CBCS SCHEME

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15ME42

## Fourth Semester B.E. Degree Examination, Jan./Feb.2021 Kinematics of Machinery

Time: 3 hrs.

Max. Marks: 80

**Note: Answer any FIVE full questions, choosing ONE full question from each module.**

### Module-1

- 1 a. Explain :  
 (i) Kinematic chain      (ii) Mechanism      (iii) Degrees of freedom. (06 Marks)  
 b. Explain with neat sketches:  
 (i) Whit worth quick return motion mechanism.      (ii) Toggle mechanism. (10 Marks)

OR

- 2 a. Explain :  
 (i) Grubler's criterion. (02 Marks)  
 (ii) Sketch and explain inversions of Grashoff's chain. (09 Marks)  
 b. Sketch and explain Pantograph mechanism. (05 Marks)

### Module-2

- 3 Fig. Q3 shows configuration diagram of an engine mechanism. The dimensions are the following : Crank OA = 200 mm, Connecting rod AB = 600 mm; Distance of centre of mass from Crank end AD = 200 mm. At the instant, the crank has an angular velocity of 50 rad/s clockwise and an angular acceleration of 800 rad/s<sup>2</sup>. Calculate the  
 (i) Velocity of 'D' and angular velocity of AB.  
 (ii) Acceleration of 'D' and angular acceleration of 'AB'. (16 Marks)

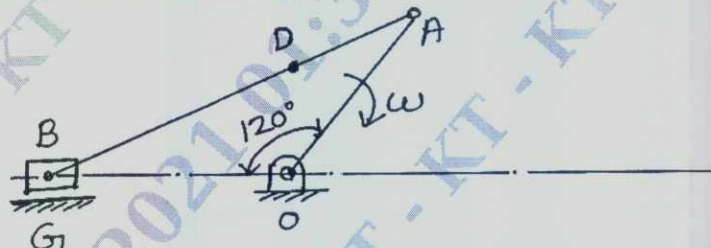


Fig. Q3

OR

- 4 a. Explain Klein's construction for slider crank mechanism to find velocity and acceleration of piston. (06 Marks)  
 b. In a slider crank mechanism the length of the crank and connecting rod are 200 mm and 800 mm respectively. Locate all the I-centres of the mechanism for the position of the crank when it has turned 30° from the inner dead centre. Also find the velocity of the slider and the angular velocity of the connecting rod if the crank rotates at 40 rad/s. (10 Marks)

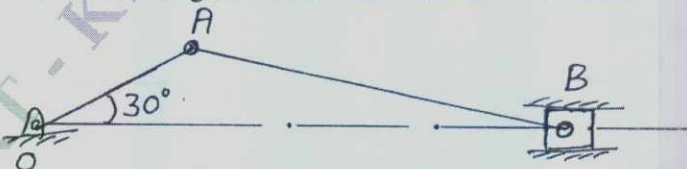


Fig. Q4 (b)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.



Module-3

- 5 In a four bar mechanism ABCD, link AB = 300 mm, BC = 360 mm, CD = 360 mm and the fixed link AD = 600 mm; The angle BAD = 60°, the link AB has an angular velocity of 10 rad/sec and an angular acceleration of 30 rad/sec<sup>2</sup>, both clockwise. Determine the angular velocity and angular acceleration of link BC and CD by using complex algebra method. (16 Marks)

OR

- 6 a. Derive Freudensteins equation for four bar mechanism. (10 Marks)  
b. Explain function generation for Four bar mechanism, by any method (two position synthesis). (06 Marks)

Module-4

- 7 a. Derive an expression for minimum number of teeth to avoid interference on a gear wheel. (06 Marks)  
b. Two involute gear wheels having module 3 mm and pressure angle 20° mesh externally to give a velocity ratio of 3. The pinion rotates at 75 rpm and addendum is equal to one module. Determine (i) The number of teeth on each wheel to avoid interference (ii) The length of path and arc of contact (iii) The number of pairs of teeth in contact. (10 Marks)

OR

- 8 a. Define gear trains. Explain different types of gear trains. (07 Marks)  
b. An epicyclic gear train is shown in Fig. Q8 (b). The number of teeth on A and B are 80 and 200 respectively. Determine the speed of the arm 'a',  
(i) If A rotates at 100 rpm clockwise and B at 50 rpm counter clockwise.  
(ii) If A rotates at 100 rpm clockwise and B is stationary. (09 Marks)

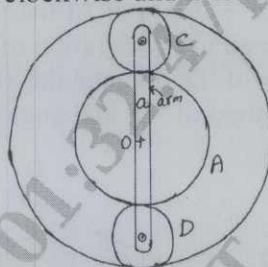


Fig. Q8 (b)

Module-5

- 9 The following data relate to a cam profile in which the follower moves with uniform acceleration and deceleration during ascent and descent:  
Minimum radius of cam = 25 mm, Roller diameter = 7.5 mm, Lift = 28 mm, Offset of the follower axis = 12 mm towards right angle of ascent = 60°, Angle of descent = 90°, Angle of dwell between ascent and descent = 45°, Speed of the cam = 200 rpm. Draw the profile of the cam and determine the maximum velocity and uniform acceleration of the follower during outstroke. (16 Marks)

OR

- 10 a. Define the terms:  
(i) Cam profile (ii) Pressure angle  
(iii) Trace point (iv) Pitch circle. (04 Marks)  
b. Derive expressions for displacement, velocity and acceleration of the follower when the flat faced follower touching circular flank (Arc cam). (12 Marks)

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# CBCS SCHEME

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15ME43

Fourth Semester B.E. Degree Examination, Jan./Feb.2021

## Applied Thermodynamics

Time: 3 hrs.

Max. Marks: 80

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. Use of Thermodynamics data hand book permitted.

### Module-1

- 1 a. Derive an expression for mean effective pressure of an Otto cycle. (08 Marks)  
b. The pressures on the compression curve of a diesel engine are at  $\frac{1}{8}$  of stroke 1.4 bar and at  $\frac{7}{8}$  stroke 14 bar. Estimate the compression ratio, calculate the air standard efficiency and mean effective pressure of the engine if the cut-off occurs at  $\frac{1}{15}$  of the stroke. Assume initially air is at 1 bar and 27°C. (08 Marks)

OR

- 2 a. For a gas turbine working on ideal Brayton cycle, show that the maximum network produced can be expressed in terms of maximum and minimum temperatures in the cycle as,  
$$[W_{net}]_{max} = C_p (\sqrt{T_{max}} - \sqrt{T_{min}})^2$$
 (08 Marks)  
b. In a reheat gas turbine cycle comprising one compressor and two turbines, air is compressed from 1 bar 27°C to 6 bar. The highest temperature in the cycle is 900°C. The expansion in the first stage turbine is such that the work from it just equals the work required by the compressor. Air is reheated between the two stages of expansion to 850°C. Assume that the isentropic efficiency of the compressor, the first stage and second stage turbines are 85% each and that working substance is air. Calculate the cycle efficiency. (08 Marks)

### Module-2

- 3 a. Discuss the effect of, (i) Boiler pressure (ii) Condenser pressure (iii) Superheat on the performance of Rankine cycle. (08 Marks)  
b. Steam at 20 bar, 360°C is expanded in a steam turbine to a pressure of 0.08 bar. If then enters a condenser, where it is condensed to saturated liquid water. Assuming the turbine and feed pump efficiencies as 60% and 90% respectively, determine per kg of steam, the network, the heat transferred to the working fluid and the Rankine efficiency of the cycle. (08 Marks)

OR

- 4 a. With neat sketch, explain working of binary vapour cycle. (08 Marks)  
b. In a reheat cycle, steam at 150 bar, 500°C expands in HP turbine till it is saturated vapour. It is then reheated at constant pressure to 400°C and then expanded in LP turbine to 40°C. If the maximum moisture content at the turbine exhaust is limited to 15%, find (i) The Reheat pressure (ii) The pressure of steam at the inlet to HP turbine (iii) The cycle efficiency. (08 Marks)



**Module-3**

- 5 a. Define : (i) Stoichiometric air, (ii) Enthalpy of formation  
(iii) Enthalpy of reaction, (iv) Adiabatic flame temperature. (08 Marks)
- b. The products of combustion of an unknown hydrocarbon  $C_xH_y$  have following composition as measured by an orset apparatus:  $CO_2 = 8\%$ ,  $O_2 = 8.8\%$ ,  $CO = 0.9\%$ ,  $N_2 = 82.3\%$ , Determine (i) Composition of fuel (ii) Air fuel ratio (iii) Percentage of excess air used. (08 Marks)

**OR**

- 6 a. With a P- $\theta$  diagram, explain stages of combustion SI engine. (08 Marks)
- b. Morse test is conducted on 4-S four cylinder petrol Engine at constant speed and the following power is measured: with all cylinders working = 15.6 kW. With number 1 cylinder cut-off = 11.1 kW, with number 2 cylinder cut off = 11.3 kW. With number 3 cylinder cut-off = 10.8 kW. With number 4 cylinder cut off = 11.0 kW. The bore and stroke of each cylinder is 75 mm and 100 mm respectively. The clearance volume of the cylinder is 100 cc. The fuel is consumed at the rate 6 kg/hr. If the calorific value of the fuel is 42000 kJ/kg. Determine (i) Indicated power (ii) Frictional power (iii) Mechanical efficiency (v) Relative efficiency with respect to brake thermal efficiency. (08 Marks)

**Module-4**

- 7 a. Sketch the flow diagram and the corresponding pressure volume diagram of an air refrigeration working on ideal Bell Coleman cycle and also derive an expression for COP of Bell Coleman cycle. (08 Marks)
- b. An air refrigeration system is to be designed according to the following specifications:  
Pressure of air at compressor inlet = 101 KPa.  
Pressure of air at compressor outlet = 404 KPa.  
Temperature of air at compressor inlet =  $-6^\circ C$   
Temperature of air at turbine inlet =  $27^\circ C$ .  
Isentropic efficiency of compressor = 85%  
Isentropic efficiency of turbine = 85%  
Determine:  
(i) COP of the cycle  
(ii) Power required to produce 1 ton of refrigeration (iii) Mass flow rate of air required for 1 ton refrigeration. (08 Marks)

**OR**

- 8 a. Define the following terms : (i) Wet Bulb Temperature (WBT). (ii) Specific Humidity (SH)  
(iii) Relative Humidity (RH) (iv) Degree of Saturation (DS). (08 Marks)
- b. An air conditioning system is designed under the following conditions : Outdoor conditions :  $30^\circ C$  DBT, 75% RH. Required conditions :  $22^\circ C$  DBT, 70% RH, Amount of air circulated  $3.33 \text{ m}^3/\text{sec}$ . Coil Dew Point Temperature (DPT) =  $14^\circ C$ . The required conditions is achieved first by cooling and humidification and then by heating. Estimate (i) The capacity of the cooling coil in tones of refrigeration. (ii) Capacity of heating coil in kW (iii) The amount of water vapour removed in kg/hr. (08 Marks)

**Module-5**

- 9 a. Obtain an expression for the volumetric efficiency of a single stage air compressor in terms of pressure ratio, clearance and 'n' the exponent of compressions expansion. (06 Marks)
- b. Why Intercooling is necessary in multistage compression? (02 Marks)

- c. A single stage double acting air compressor is required to deliver  $14 \text{ m}^3$  of air per minute measured at 1.013 bar and  $15^\circ\text{C}$ . The delivery pressure is 7 bar and the speed is 300 rpm. Take the clearance volume is 5% of swept volume with a compression and re expansion index of  $n = 1.3$ . Calculate the swept volume of the cylinder, the delivery temperature and the indicated power. (08 Marks)

OR

- 10 a. With neat sketch, explain different shapes of nozzle. (03 Marks)
- b. Derive an expression for exit velocity of nozzle in terms of pressure ratio and index of expansion. (05 Marks)
- c. A multistage air compressor compresses air from 1 bar to 40 bar. The maximum temperature of air not to exceed 400 K in any stage. If the law of compression is  $PV^{1.3} = C$ . Find number of stages for minimum power input, also find the actual intermediate pressure and temperatures. What will be minimum power input in kW required to compress and deliver 10 kg/min of air and the rate of heat rejection in each inter cooler. Assume ambient temperature =  $27^\circ\text{C}$  and perfect inter cooling between stages. (08 Marks)

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15ME44

## Fourth Semester B.E. Degree Examination, Jan./Feb. 2021 Fluid Mechanics

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Differentiate between Newtonian and non-Newtonian fluids. Give examples for each. (04 Marks)
- b. Define surface tension of a liquid. Derive an expression for surface tension of a  
i) liquid droplet ii) hollow bubble. (05 Marks)
- c. A 15cm diameter vertical cylinder rotates concentrically inside another cylinder of 15.1cm diameter. Both cylinders are 25cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12N-m is required to rotate the inner cylinder at 100rpm, determine the viscosity of the fluid. (07 Marks)

OR

- 2 a. Derive an expression for the depth of centre of pressure from free surface of liquid of vertical plane surface submerged in the liquid. (08 Marks)
- b. A solid cylinder of diameter 4m has a height of 4m. Find the metacentric height of the cylinder if the specific gravity of the material of the cylinder is 0.6 and it is floating in water with its axis vertical. State whether the equilibrium is stable or unstable. (08 Marks)

### Module-2

- 3 a. Define the following :  
i) Steady and unsteady flow  
ii) Laminar and Turbulent flow. (04 Marks)
- b. Derive the continuity equation in three dimensional Cartesian coordinate for a steady incompressible fluid flow. (06 Marks)
- c. The velocity components in a two dimensional flow field for an incompressible fluid are expressed as  $u = \frac{y^3}{3} + 2x - x^2y$ ,  $v = xy^2 - 2y - \frac{x^3}{3}$ . Obtain an expression for velocity potential function. (06 Marks)

OR

- 4 a. With usual notations, show that the discharge through a venturimeter is given by  
$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$
 (06 Marks)
- b. A pipeline carrying oil of specific gravity 0.9 changes in diameter from 20cm at a position A to 50cm at position B which is 5m at higher level. If the pressure at A and B are 10N/cm<sup>2</sup> and 6N/cm<sup>2</sup> respectively and discharge is 200litres/s, determine the loss of head and the direction of flow. (10 Marks)

Module-3

- 5 a. Prove that the velocity distribution across the cross section of a circular pipe during viscous fluid flow is parabolic in nature. Also prove that the maximum velocity is in the centre of the pipe and is equal to twice the average velocity. (10 Marks)
- b. Water at 15°C flows between two large parallel plates at a distance of 1.6mm apart. Determine : i) the maximum velocity ii) the pressure drop per unit length iii) the shear stress at the walls of the plates if the average velocity is 0.2m/s. The viscosity of water at 15°C is given as 0.01 poise. (06 Marks)

OR

- 6 a. Derive an expression for loss of head due to sudden enlargement of a pipe. (08 Marks)
- b. An oil of specific gravity 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200mm at the rate of 60 liters/s. Find the head loss due to friction for a 500m length of the pipe. Find the power required to maintain this flow. (08 Marks)

Module-4

- 7 a. Find the displacement thickness and the momentum thickness for the velocity distribution in the boundary layer given by  $\frac{u}{U_\infty} = \frac{y}{\delta}$ , where  $u$  is the velocity at a distance  $y$  from the plate and  $u = U_\infty$  and  $y = \delta$ , where  $\delta$  = boundary layer thickness. Also calculate the ratio of displacement thickness to momentum thickness. (08 Marks)
- b. Experiments were conducted in a wind tunnel with a wind speed of 50km/h on a flat plate 2m long and 1m wide. The density of air is 1.15kg/m<sup>3</sup>. The plate is kept at such an angle that the coefficient of lift and drag are 0.75 and 0.15 respectively. Determine :  
i) lift force ii) drag force iii) resultant force iv) power exerted by air stream on the plate. (08 Marks)

OR

- 8 a. Explain the following terms :  
i) Geometric similarity ii) Kinematic similarity iii) Dynamic similarity. (06 Marks)
- b. A partially submerged body is towed in water. The resistance  $R$  to its motion depends on the density  $\rho$ , the viscosity  $\mu$  of water, length  $l$  of the body, velocity  $v$  of the body and the acceleration due to gravity  $g$ . By using Buckingham's  $\pi$ -theorem, show that the resistance to the motion can be expressed in the form

$$R = \rho l^2 v^2 \phi \left[ \frac{\mu}{\rho v L}, \frac{lg}{v^2} \right] \quad (10 \text{ Marks})$$

Module-5

- 9 a. From fundamentals, show that the velocity of a sound wave in a compressible fluid is given by  $C = \sqrt{\frac{dp}{d\rho}}$ . Further, show that this sonic velocity for an isentropic medium is given by  $c = \sqrt{rRT}$ , where  $r$  = ratio of specific heats,  $R$  = Gas constant ;  $T$  = Temperature. (08 Marks)
- b. Define Mach number. Explain its significance in compressible flow. (04 Marks)
- c. Compute the velocity of a bullet fired in still air and Mach number when the mach angle is 30°. Take  $R = 0.28714 \text{kJ/kg K}$  and  $r = 1.4$ . Assume air temperature to be 15°C. (04 Marks)

OR

- 10 a. Define and write the expression for : i) Stagnation enthalpy ii) Stagnation temperature iii) Stagnation pressure. (06 Marks)
- b. Mention the advantages, disadvantages and limitation of CFD. (10 Marks)





Module-3

- 5 a. Find Laplace transform of  $\cos^3 at$ . (06 Marks)
- b. A periodic function of period  $2a$  is defined by
- $$f(t) = \begin{cases} E & \text{for } 0 \leq t \leq a \\ -E & \text{for } a \leq t \leq 2a \end{cases}$$
- where  $E$  is a constant. Find  $L\{f(t)\}$ . (05 Marks)
- c. Express the function  $f(t) = \begin{cases} \cos t, & t \leq \pi \\ \sin t, & t > \pi \end{cases}$  in terms of unit step function and hence find its Laplace transform. (05 Marks)

OR

- 6 a. Find  $L\left\{\frac{\cos at - \cos bt}{t}\right\}$  (06 Marks)
- b. Find  $L\{\sin t \sin 2t \sin 3t\}$  (05 Marks)
- c. Express the function  $f(t) = \begin{cases} t^2, & t \leq 2 \\ 4t, & t > 2 \end{cases}$  in terms of unit step function and hence find its Laplace transform. (05 Marks)

Module-4

- 7 a. Find  $L^{-1}\left\{\frac{2s+3}{s^3-6s^2+11s-6}\right\}$  (06 Marks)
- b. Find  $L^{-1}\left\{\cot^{-1}\left(\frac{s}{a}\right)\right\}$  (05 Marks)
- c. Using Laplace transform method, solve the initial value problem  $y'' + 5y' + 6y = 5e^{2t}$ , given that  $y(0) = 2$  and  $y'(0) = 1$ . (05 Marks)

OR

- 8 a. Find  $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$  (06 Marks)
- b. Find  $L^{-1}\left\{\log\left(\frac{s^2+1}{s(s+1)}\right)\right\}$  (05 Marks)
- c. Using Laplace transforms, solve the initial value problem  $y' + y = \sin t$ , given that  $y(0) = 0$ . (05 Marks)

Module-5

- 9 a. For any two events  $A$  and  $B$ , prove that  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  (06 Marks)
- b. If  $A$  and  $B$  are any two events with  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{1}{3}$  and  $P(A \cap B) = \frac{1}{4}$ , find  $P(A/B)$ ,  $P(B/A)$ ,  $P(\bar{A}/\bar{B})$  and  $P(\bar{B}/\bar{A})$  (05 Marks)
- c. From 6 positive and 8 negative numbers, 4 numbers are selected at random and are multiplied. What is the probability that the product is positive? (05 Marks)

OR

- 10 a. State and prove Baye's theorem. (06 Marks)
- b. A book shelf contains 20 books of which 12 are on electronics and 8 are on mathematics. If 3 books are selected at random, find the probability that all the 3 books are on the same subject. (05 Marks)
- c. The machines  $A, B, C$  produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. If an item is selected at random is found to be defective, then determine the probability that the item was manufactured by machine  $A$ . (05 Marks)